QUIZ #1 M427L SUMMER 2005: SPENCER STIRLING (PROF. CHAS FRIEDMAN)

1. (10 points). Find the equation for the tangent plane at the point $(3, 5, -4)$ of the hyperboloid $x^2 + y^2 - z^2 = 18$.

2. (5 points). Write the equation for the plane in (1) translated and shifted appropriately such that the plane passes through the origin instead of the point specified in (1).

3. (10 points). Find the unit normal to the plane in (1) (actually there are two of them since you can just reverse the direction of the unit normal and get another unit normal).

4. (5 points). Assume $z(x, y)$ (as in problem 1) describes the elevation of a snow-covered mountain. In which direction will a hapless mountaineer (sitting at the coordinates $(3, 5, -4)$) slide to his grisly death (leave the length of the vector in so that I can tell how fast he will succomb to a shallow grave of ice)?

5. (extra credit 10 points). Fortunately for our mountaineer the surveyor at the USGS used faulty equipment designed by an engineer who *failed vector calculus.* Taking another survey of the land costs $20 million, so instead the USGS decided to hire a certain mathematical physicist at UT for exactly half that amount to fix the problem (a good bargain).

This young go-getter determined the exact faults in the equipment and found that by stretching the map the right way he could produce the correct elevation map. This stretching is described by the equation $g(x, y) = (e^{x+y}, x^2 + y^2)$. In other words, the function $z \circ g$ gives the correct elevation. Now determine the vector encoding the steepest *descent* like you did in the previous problem (assume that the mountaineer's position AFTER THE STRETCH is unchanged - so after the stretch he sits at $(3, 5)$).