

HOMEWORK #5 SELECTED SOLUTIONS (M427K FALL 2004)

INTRODUCTION

The point of this set of exercises is to remind you how to derive the necessary trig identities *and* to show how mathematicians approach things.

You are supposed to solve these problems using *ONLY* the following given information (meaning this is stuff you should memorize, and everything else comes from it)

$$(0.1) \quad \sin x \text{ and } \cos x \text{ are NOT } \equiv 0$$

$$(0.2) \quad \sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$(0.3) \quad \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

1. EXPLAIN EQUATION 0.1

1.1. Solution. This problem is just trying to get you to know the difference between the \equiv sign and the $=$ sign. If I write $f(x) \equiv 0$, this just means that $f(x)$ is zero *everywhere*. It a definition. I'm just saying that I define f to be zero for every x .

If I write $f(x) = 0$ then I am writing an equation to be solved. This means that I am trying to find all values of x such that $f(x)$ is zero.

For example, if I write $\sin(x) = 0$, then the solutions are $x = 0, \pi, \dots$. If I write $\sin x \equiv 0$ then I am writing something wrong because I know damned well that there are values of x such that the sine doesn't vanish... i.e. $\sin 0.76 = 0.68$. So equation 0.1 is correct.

2. SHOW THAT $\sin 0 = 0$ AND $\cos 0 = 1$

2.1. Solution to my claim that $\sin 0 = 0$.

Proof. Notice that $0 - 0 = 0$, so $\sin 0 = \sin(0 - 0)$. Using equation 0.2 this is just $\sin(0 - 0) = \sin 0 \cos 0 - \sin 0 \cos 0 = 0!$ Voila. Pretty slick, huh? \square

$$3. \cos(x) = \cos(-x), \sin(x) = -\sin(-x), \tan(x) = -\tan(-x)$$

3.1. Solution to my claim that $\cos(x) = \cos(-x)$.

Proof. From the previous problem I know that $\cos 0 = 1$ and that $\sin 0 = 0$, so I have using equation 0.3

$$\cos(-x) = \cos(0 - x) = \cos 0 \cos x + \sin 0 \sin x = \cos x.$$

That's it!!! \square

4. $\cos^2 x + \sin^2 x = 1$, AND OTHER STUFF4.1. **Solution to my claim that $\cos^2 x + \sin^2 x = 1$.**

Proof. This one is almost exactly like the last one. From problem 2 I know that $\cos 0 = 1$, so I have using equation 0.3

$$1 = \cos(0) = \cos(x - x) = \cos x \cos x + \sin x \sin x.$$

Boom!!! Try these same techniques on the rest of the problems. Note that once you prove something, you don't need to prove it again. You can just add it to your toolbox and refer to it. \square