HOMEWORK #17 (M427K FALL 2004)

Introduction

Use power series expansion to solve the differential equation. This is an important technique that you will see many times in applications (for example, quantum mechanics)

1. Solve the DiffEq Using Infinite Series

$$y''(x) + 16y(x) = 0$$

1.1. **Hint.** The solution is going to be *some* function (maybe we don't even have a name for it), and we know that ANY "sufficiently good" function can be written as a power series (taylor series). So let's just propose a solution in the form of a power series. So I guess

$$y(x) = \sum_{n=0}^{\infty} a_n x^n.$$

Now I plug this into the differential equation and I collect "like terms". Since the RHS=0, I know that each term separately in the infinite series must be =0. This will give me a recursive formula for each of the a_n's.

So then that's my solution. I just figure out the a_n's and that's it. Now, at the end, maybe I recognize the power series

In this situation, I will actually get two different recursive formulas: one for the even a_n's, and one for the odd ones. So they are pretty much independent, and I can solve for them separately. Note that this won't always happen.

Anyway, one of the power series (with even terms) will be the cosine, and the other one (with odd terms) will be the sine. This is exactly what we expect from our other methods of solving this diffeq.