SOLUTIONS

M1090 midterm 2 (Spencer Stirling) - April 2, 2010

Directions: Use both the front and back of the paper for your solutions. You may attach more sheets if necessary. SHOW ALL WORK and CLEARLY mark your solutions.

1) (~~4 points~~) Find the inverse A^{-1} of the matrix

2 pts extra credit

$$\begin{pmatrix} 6 & 0 & 1 \\ -3 & 5 & -3 \\ 7 & 3 & 6 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 6 & 0 & 1 & 1 & 0 & 0 \\ -3 & 5 & -3 & 0 & 1 & 0 \\ 7 & 3 & 6 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\substack{1 + 2 \cdot 2 \to 2 \\ -7 \cdot 1 + 6 \cdot 3 \to 3}]{\text{2 steps in 1}} \left(\begin{array}{ccc|ccc} 6 & 0 & 1 & 1 & 0 & 0 \\ 0 & 10 & -5 & 1 & 2 & 0 \\ 0 & 18 & 29 & -7 & 0 & 6 \end{array}\right)$$

$$\xrightarrow{18 \cdot 2 + -10 \cdot 3 \to 3} \left(\begin{array}{ccc|ccc} 6 & 0 & 1 & 1 & 0 & 0 \\ 0 & 10 & -5 & 1 & 2 & 0 \\ 0 & 0 & -380 & 88 & 36 & -60 \end{array}\right) \xrightarrow{\frac{3}{4} \to 3} \left(\begin{array}{ccc|ccc} 6 & 0 & 1 & 1 & 0 & 0 \\ 0 & 10 & -5 & 1 & 2 & 0 \\ 0 & 0 & -95 & 22 & 9 & -15 \end{array}\right)$$

$$\xrightarrow{-19 \cdot 2 + 3 \to 2} \left(\begin{array}{ccc|ccc} 6 & 0 & 1 & 1 & 0 & 0 \\ 0 & -190 & 0 & 3 & -29 & -15 \\ 0 & 0 & -95 & 22 & 9 & -15 \end{array}\right) \xrightarrow{95 \cdot 1 + 3 \to 1} \left(\begin{array}{ccc|ccc} 570 & 0 & 0 & 117 & 9 & -15 \\ 0 & -190 & 0 & 3 & -29 & -15 \\ 0 & 0 & -95 & 22 & 9 & -15 \end{array}\right)$$

$$\xrightarrow{\text{divide all rows}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{117}{570} & \frac{9}{570} & \frac{-15}{570} \\ 0 & 1 & 0 & -3/190 & 29/190 & 15/190 \\ 0 & 0 & 1 & -22/95 & -9/95 & \end{array}\right)$$

so

$$\boxed{A^{-1} = \begin{pmatrix} 39/190 & 3/190 & -5/190 \\ -3/190 & 29/190 & 15/190 \\ -44/190 & -18/190 & 30/190 \end{pmatrix}}$$

1 pt extra credit

2) ~~(3 points)~~ Solve the following system of linear equations (hint: the previous problem is useful here, however this problem can also be solved independently)

$$6x + z = 5$$
$$-3x + 5y - 3z = 1$$
$$7x + 3y + 6z = 3$$

$$\underbrace{\begin{pmatrix} 6 & 0 & 1 \\ -3 & 5 & -3 \\ 7 & 3 & 6 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$$

so $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = \frac{1}{190} \begin{pmatrix} 39 & 3 & -5 \\ -3 & 29 & 15 \\ -44 & -18 & 30 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$

$$= \frac{1}{190} \begin{pmatrix} 195 + 3 - 15 \\ -15 + 29 + 45 \\ -220 - 18 + 90 \end{pmatrix} = \boxed{\begin{pmatrix} 183/190 \\ 59/190 \\ -148/190 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}}$$

SOLUTIONS

3) (2 points) Solve the quadratic equation

$$5x^2 - 3x + 2 = 0$$

$$x = \frac{3 \pm \sqrt{9 - 4(5)(2)}}{2 \cdot 5} = \boxed{\frac{3 \pm \sqrt{-31}}{10} \quad \text{no solution (imaginary solution)}}$$

4) (3 points) Find the vertex, axis of symmetry, whether the parabola is concave up or concave down, and the zeros of the function (if they exist)

$$y = -4x^2 + 3x + 5$$

$$y = -4\left(x^2 - \frac{3}{4}x\right) + 5$$

$$= -4\left(\left(x - \frac{3}{8}\right)^2 - \frac{9}{64}\right) + 5$$

$$= -4\left(x - \frac{3}{8}\right)^2 + \frac{89}{16}$$

vertex $= \left(\frac{3}{8}, \frac{89}{16}\right)$

axis of symmetry: $x = \frac{3}{8}$

concave down

roots $-4x^2 + 3x + 5 = 0 \rightarrow x = \dfrac{-3 \pm \sqrt{9 - 4(5)(-4)}}{2(-4)}$

$$= \frac{-3 \pm \sqrt{89}}{-8} = \boxed{\frac{3 \mp \sqrt{89}}{8} = \text{roots}}$$

5) (4 points) A movie theater has found that for every 15 cent increase in ticket prices, 10 fewer people will buy tickets for that movie. When tickets are priced at \$8.00 the theater can expect to sell 400 tickets for that day. Given that the total ticket revenue equals the price of the ticket times the number of tickets sold, what is the price which will bring the maximum revenue and what is this maximum revenue (notice there are TWO questions here)?

demand slope: $\frac{\Delta q}{\Delta p} = \frac{-10}{0.15}$ ← 10 people less ← 15 cents raise

point slope formula:

$$q - 400 = \frac{-10}{0.15}(p - 8)$$

400 people; \$8.00

$$0.15q - 60 = -10(p-8)$$

$$-0.015q + 6 = p - 8 \longrightarrow p = -0.015q + 14 \quad ☆$$

Revenue $R(q) = pq = (14 - 0.015q)q = -0.015q^2 + 14q$

complete square:

$$R(q) = -0.015(q^2 - 933.\overline{33})$$

$$= -0.015\left(\left(q - \frac{933.33}{2}\right)^2 - \left(\frac{933.33}{2}\right)^2\right)$$

$$= -0.015\left((q - 466.\overline{66})^2\right) + 3266.66$$

so at $q = 466.66$ people the $\boxed{\text{Revenue will be } \$3266.66}$

to find ticket price use ☆: $p = (-0.015)(466.66) + 14 = \boxed{\$7.00 = p}$

6) (3 points) Find the number of units that need to be produced and sold to break even given the revenue and cost functions

$$R(x) = 790x - 0.5x^2$$
$$C(x) = 100 - 10x + 0.5x^2$$

$P(x) = R(x) - C(x) = 790x - 0.5x^2 - (100 - 10x + 0.5x^2)$

$= x^2 + 800x - 100$ set $= 0$ for break even

$x = \dfrac{-800 \pm \sqrt{(800)^2 - 4(1)(-100)}}{2} = \dfrac{-800 \pm 800.25}{2}$

positive # only makes sense $= \boxed{0.125 = x}$

7) (3 points) Given the supply and demand equations find the equilibrium price and quantity

supply: $p = q^2 + 12$
demand: $p = -5q^2 - 8q + 75$

$q^2 + 12 = -5q^2 - 8q + 75$

$+6q^2 + 8q - 63 = 0 \rightarrow q = \dfrac{-8 \pm \sqrt{64 - 4(6)(-63)}}{2 \cdot 6}$

$q = \dfrac{-8 \pm 39.69}{12} \xrightarrow{\text{only positive } q \text{ makes sense}} \boxed{q \approx 2.64}$

$p = (2.64)^2 + 12 = \boxed{18.97 = p}$