Key

Math1090 Final Review Exercises
(from old Final Exams)

1A) Find the inverse of the following matrix, if possible. If it's not possible, then explain why.

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

$A^{-1} =$ $\begin{bmatrix} 1 & 2/5 \\ 0 & 1/5 \end{bmatrix}$

1B) For $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix}$, perform the indicated matrix operations, if possible. If not possible, explain why.

(a) $A + A^T$

$A + A^T =$ $\begin{bmatrix} 2 & 3 & 6 \\ 3 & 4 & 1 \\ 6 & 1 & 6 \end{bmatrix}$

(b) BC

$BC =$ $\begin{bmatrix} 29 & 25 \\ 10 & 12 \end{bmatrix}$

1C) Use Gauss-Jordan Elimination to solve the following system.

$$\begin{aligned} 2x - 4y + 2z &= -4 \\ 4x - 9y + 7z &= 2 \\ -2x + 4y - 3z &= 10 \end{aligned}$$

$x =$ 36

$y =$ 16

$z =$ -6

2A) Given the arithmetic sequence

-2, 1, 4, 7, 10, ...

(a) Find the 100^{th} term.

100^{th} term = 295

(b) Find the sum of the first 100 terms.

Sum of first 100 terms = 14650

2B) How much would have to be invested at the end of each year at 6% interest compounded annually to pay off a debt of $80,000 in 10 years?

$ 6,069.44

2C) A lottery prize worth $1,000,000 is awarded in payments of $10,000 five times a year for 20 years. Suppose the money is worth 20% compounded 5 times per year.

(a) What is the interest rate, i?

i = 0.04

(b) What is the number of compoundings, n?

n = 100

(c) What is the formula used to find the present value of this prize?

____

(d) What is the present value of this prize?

$ $245,049.99

3A) For $f(x)=\sqrt{1-x}$ and $g(x)=x^2+1$

(a) State the domain for both functions.

Domain for $f(x)$ $x \leq 1$

Domain for $g(x)$ $x \in \mathbb{R}$

(b) Find $g \circ f$ and state the domain of this new function.

$(g \circ f)(x)$ = $2-x$

domain: $x \in \mathbb{R}$

(c) Find $\frac{f}{g}$.

$\frac{f}{g}(x)$ = $\frac{\sqrt{1-x}}{x^2+1}$

3B) Solve the equation.

$$\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1}$$

x = N.S.

3C) Find the equation of the line passing through the points (-1, 1) and (2, 3).

$y=\frac{2}{3}x+\frac{5}{3}$

4A) Graph the linear inequality.

$$-4x < 6y$$

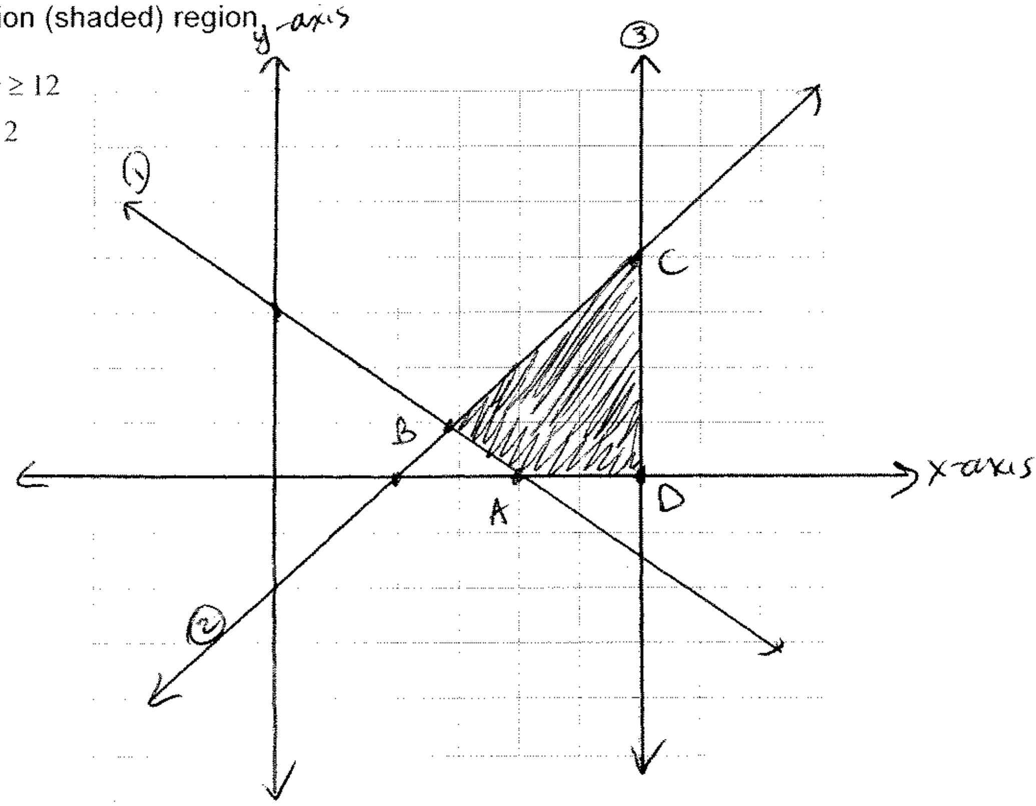
4B) Graph the system of inequalities and shade the solution region. Label all vertices for the solution (shaded) region.

① $3x + 4y \geq 12$

② $x - y \geq 2$

③ $x \leq 6$

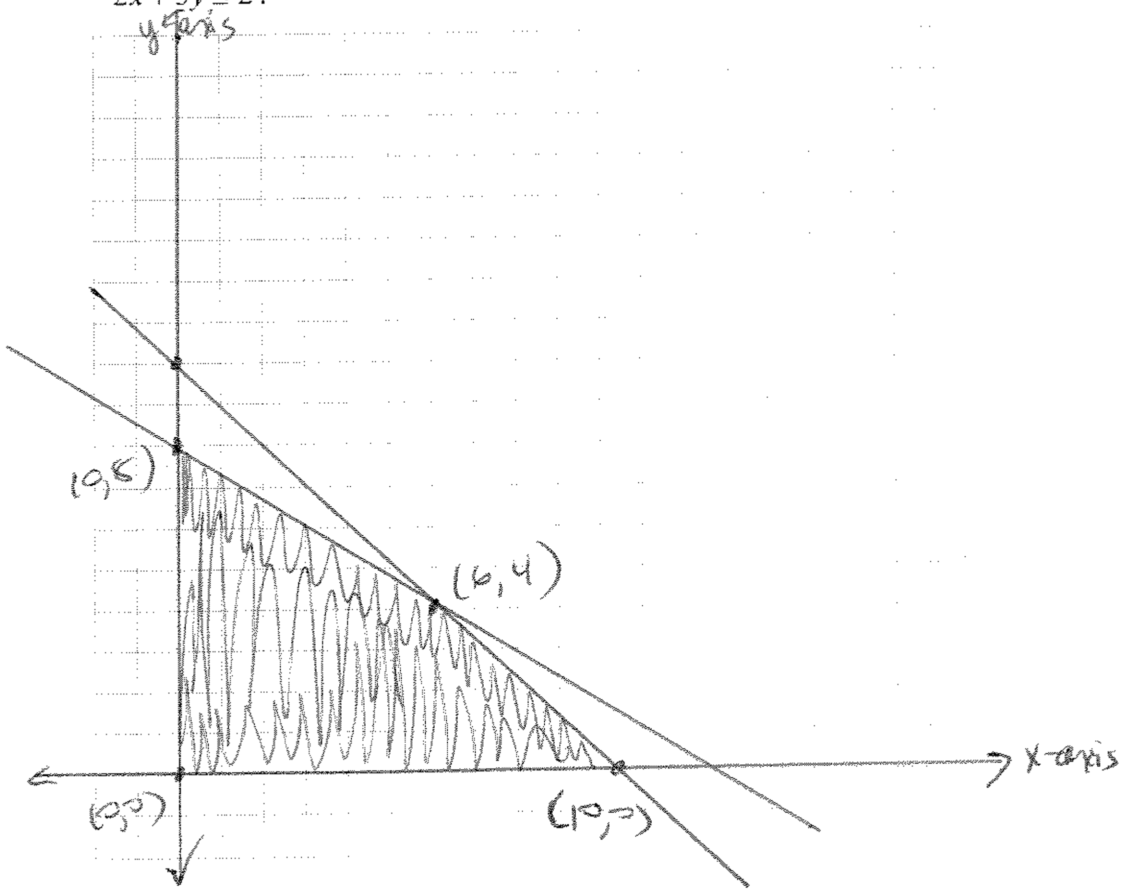
$y \geq 0$



Vertices: A(4,0) B(20/7, 6/7) C(6,4) D(6,0)

4C) Find the maximum of the objective function $f(x, y) = 2x + y$ subjected to the following constraints.

$$x \geq 0$$
$$y \geq 0$$
$$x + y \leq 10$$
$$2x + 3y \leq 24$$



Maximum value: 20 at point (10,0)

5A) If the cost of production for a product is given by $C(x) = x^2 + 11x + 84$ and the revenue is given by $R(x) = 30x$,

(a) Find the profit function $P(x)$.

$P(x) =$ $-x^2 + 19x - 84$

(b) Find the break-even point(s).

Break-even point(s): (12, 0) (7, 0)

5B) If 100 feet of fence is used to fence in a rectangular yard, then the resulting area is given by $A(x) = x(50 - x)$
where x feet is the width of the rectangle and $(50 - x)$ feet is the length. Determine the width and length that give the maximum area.

Width for max area = 25 ft

Length for max area = 25 ft

5C) Let $f(x) = -(x-1)^2 + 4$.

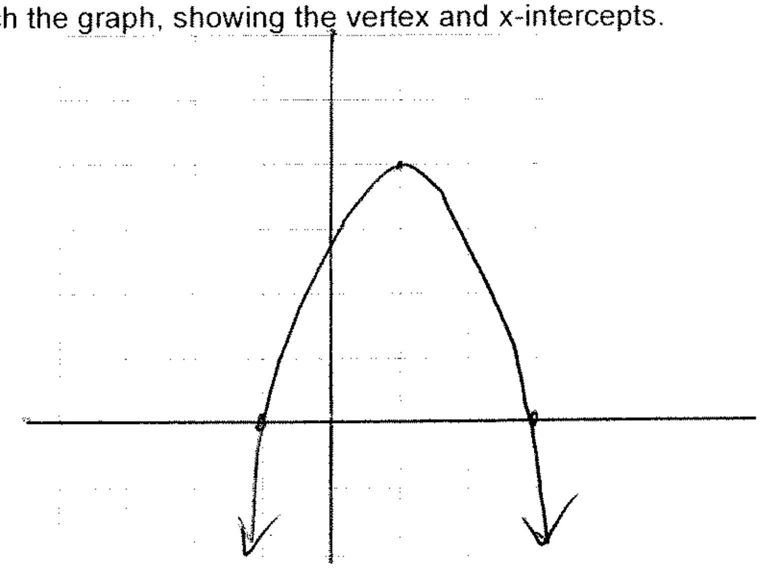
(a) Solve $f(x) = 0$ to find the x-intercepts.

x-intercepts: (3, 0) (-1, 0)

(b) Find the vertex of the parabola.

Vertex (1, 4)

(c) Sketch the graph, showing the vertex and x-intercepts.



6A) Suppose that the population of Smalltown, USA grows according to the formula

$$P(t) = 3200e^{0.025t}$$

where time t is measured in years.

(a) What is the initial population of the town (at $t = 0$)?

Initial population = 3200

(b) How long will it take the population to double?

27.7 years

(c) What is the population after 1 year? 3281

6B) Use the properties of logarithms and the fact that

$\log_{10} 2 \approx 0.3$ $\quad \log_{10} 5 \approx 0.7$ $\quad \log_{10} 7 \approx 0.85$

to find the values below.

(a) $\log_{10} 8$

$\log_{10} 8 \approx$ 0.9

(b) $\log_{10} 35$

$\log_{10} 35 \approx$ 1.55

(c) $\log_5 2$

$\log_5 2 \approx$ 3/7

6C) Rewrite $\log_2 32 = x$ in exponential form and solve for x.

Exponential form: $2^x = 32$

$x =$ 5

7. Given the matrices A, B, C and D, perform the indicated operations or state that it's not possible.

$$A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} \quad B=\begin{bmatrix}1 & 0 & 7\\ 3 & -4 & 2\end{bmatrix} \quad C=\begin{bmatrix}1 & 5 & -7 & 1\\ -1 & 6 & 2 & -4\end{bmatrix}$$

$$D=\begin{bmatrix}3 & 5\\ 1 & 2\end{bmatrix}$$

(a) AB $\begin{bmatrix}-3 & 4 & -2\\ 1 & 0 & 7\end{bmatrix}$

(b) $2A-3D$ $\longrightarrow$ $\begin{bmatrix}-9 & -17\\ -1 & -6\end{bmatrix}$

(c) D^{-1} $\begin{bmatrix}-2 & -5\\ 1 & 3\end{bmatrix}$

(d) C^{T} $\longrightarrow$ $\begin{bmatrix}1 & -1\\ 5 & 6\\ -7 & 2\\ 1 & -4\end{bmatrix}$

(e) $A+B$ not possible, sizes need to match

8. Solve the following linear system of equations, if possible.

$$\begin{aligned}3x-2y+z&=2\\ x-y+z&=2\\ 5x+10y-5z&=10\end{aligned}$$

(1, 2, 3)

9. John makes a $1000 contribution at the end of each quarter to a retirement account for 10 years earning 7% interest. After that, he makes no additional contributions and no withdrawals, and he leaves the money in the account for another 10 years.

(a) How much money is in the account after the 10 years of contributions?
$572,341.34

(b) How much money is in the account at the end of 20 years?
$1,145,596.91

10. For $f(x)=\sqrt{x-2}$ and $g(x)=x^2+1$
State the domain for both functions. $f: x \geq 2$, $g: x \in \mathbb{R}$

Find $(f+g)(5)$. $\sqrt{3}+26$

Find $(f \circ g)(x)$. $\sqrt{x^2-1}$

11. Solve the equation.

$$\frac{3x}{x-2}+\frac{1}{2}=\frac{3}{10}+\frac{6}{x-2}$$

N.S.

12. Find the equation of the line parallel to $y=-2x+1$ and passing through the point (-1, 5).

$y=-2x+3$

13. Angela buys a car. After a down payment, she still owes \$20,000. She sets up a 5-year loan with monthly payments due at the end of each month with an interest rate of 6%.

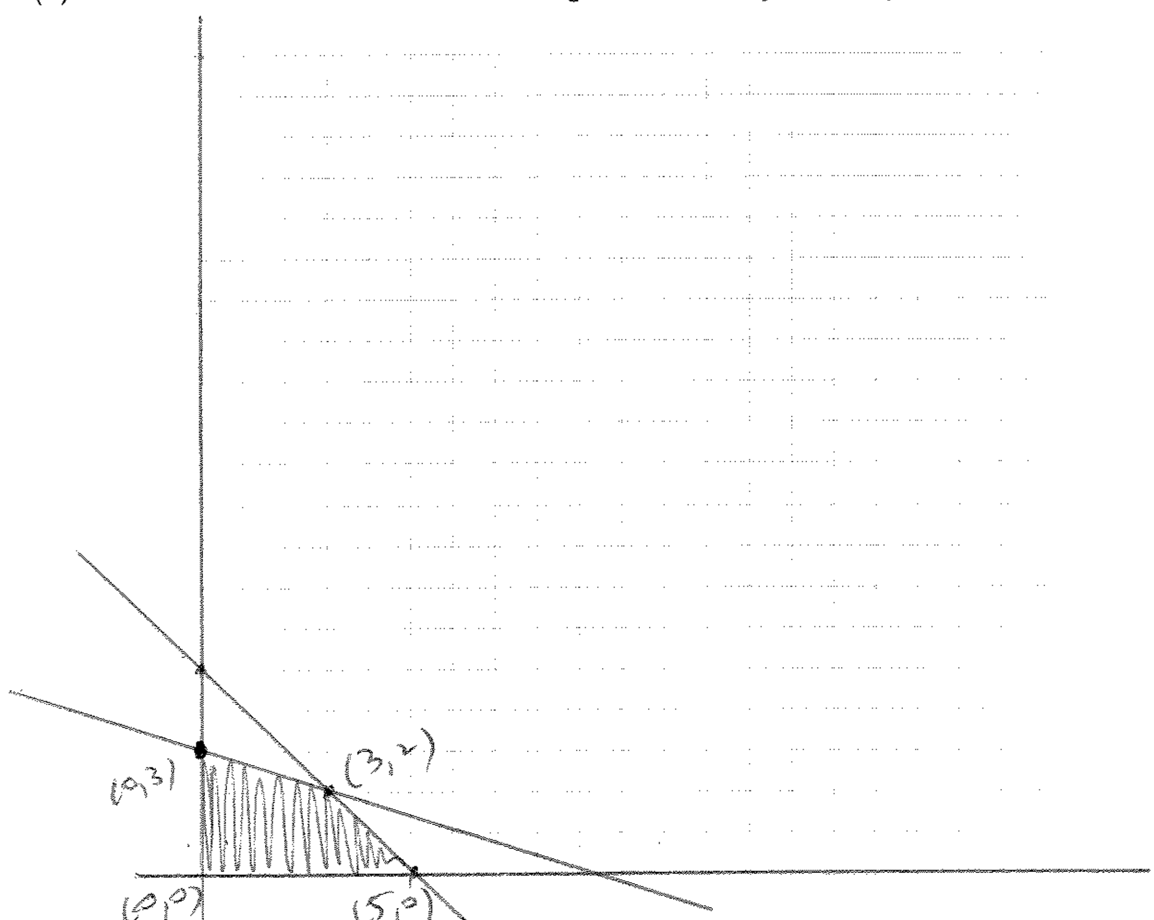
(a) How much will each monthly payment be? \$386.66

(b) If Angela decides to pay off the loan after 3 years, how much money should she pay then? \$ 8724.16

14. For the following system of inequalities

$$\begin{aligned} x+3y &\le 9 \\ 2x+2y &\le 10 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

(a) Sketch and shade in the solution region defined by the inequalities.



(b) Find the maximum of the objective function $f(x, y) = 2x + y$ subjected to the above constraints.

Maximum value: 10 at point (5,0)

15. The total costs for a company to produce and sell x units of a product are given by $C(x)=500+50x+x^2$ (in dollars). The sale price for one item is $250.

(a) Find the revenue function, $R(x)$. $R(x) = 250x$

(b) Find the profit function, $P(x)$. $P(x) = -x^2 + 200x - 500$

(c) Find the break-even point(s). $(2.532, 0)$ $(197.478, 0)$

(d) Find the number of items sold to get the maximum profit. 100

16. The population of Mathville was 12,000 in 1960 and 21,000 in 1980. The population growth of the city follows the formula

$$P(t)=P_0 e^{ht}$$

where t is the number of years after 1960.

(a) Determine P_0 and h. $P_0 = 12000$ $h = 0.02798$

(b) Estimate the population of Mathville in the year 2000. 36750

(c) How many years after 1960 will the population grow to be 34,000? ~ 37 years

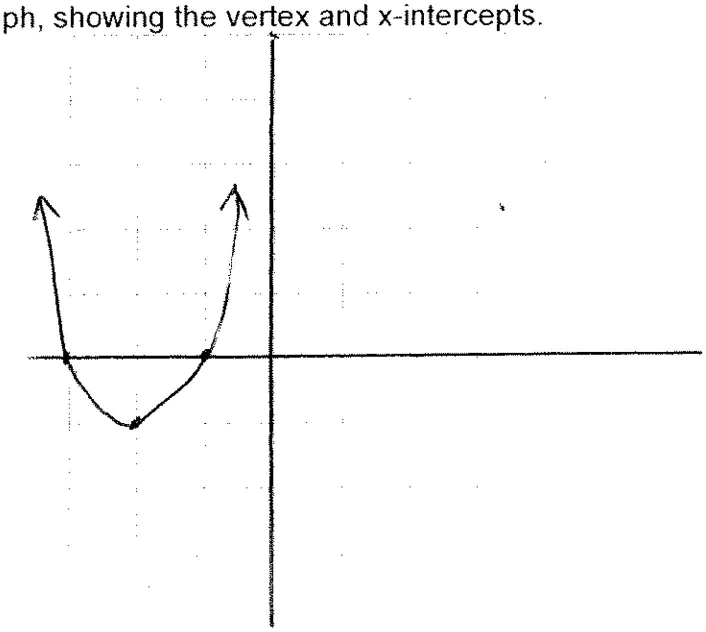
17. Let $y=x^2+4x+3$.

(a) Find the vertex of the parabola. $(-2, -1)$

(b) Tell if it's a minimum or maximum point. min.

(c) Solve $y = 0$ to find the x-intercepts, if there are any. $(-1, 0)$ $(-3, 0)$

(d) Sketch the graph, showing the vertex and x-intercepts.



18. Solve for the exact value of x.

$$\log_3(x-2)+\log_3 5=3$$ $x = \frac{37}{5}$

19. Solve for x. $|3-4x|=13$. $x = 4, -5/2$

20. The Utah Company manufactures a certain product that has a selling price of $40 per unit. Fixed costs are $1,600 and variable costs are $20 per unit. Determine the least number of units that must be sold for the company to have a profit of no less than $5,000. [All work must be shown; the guess-and-test method is not acceptable.]

330

21. A rectangular plot of land has an area of 18,000 square feet. If its length is five times its width, how much fencing would be required to surround the property?

720 ft

22. For the following functions, answer the specified questions.

$$f(x)=\frac{x+1}{3x^2+20x+25} \qquad g(x)=-x$$

(a) What is the domain of $f(x)$? $x \in \mathbb{R},\ x \neq -5, -5/3$

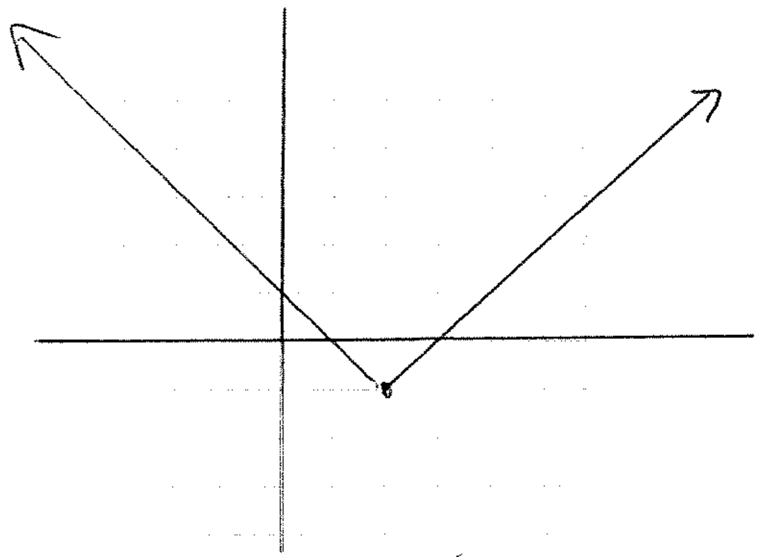
(b) What is the domain of $g(x)$? $x \in \mathbb{R}$

(c) $f(-1)$ = 0

(d) $f(0)$ = 1/25

(e) $(f \circ g)(2)$ = 1/3

23. Graph the function $y=|x-2|-1$ and determine the x- and y-intercepts.



x-intercept: (3,0) (1,0)

y-intercept: (0,1)

24. The students at a university buy 3,000 graphing calculators per year when they cost $50 each, and they buy 2,000 calculators per year when they cost $100 each. Let P be the price per calculator and Q be the quantity of calculators sold. Assuming the relationship between P and Q is linear, give an equation expressing P in terms of Q. $P = -\frac{1}{20} Q + 200$

25. Find the value for x which maximizes the quadratic function
$f(x)=-x^2+11x-24$. $11/2$

26. Solve the following equations.

(a) $\ln(2x+7)=0$ $x = -3$

(b) $e^{2x}=9$ $x = \frac{1}{2} \ln 9$

(c) $\log x+\log 3=2$ $x = \frac{100}{3}$

27. Jeremy wants to make one savings deposit today so that in 7 years, he will have $16,000. Given an interest rate of 4% compounded semiannually (twice a year), how much money should Jeremy deposit?

$12,126.00

28. Brittany is 25 years old and she plans to retire when she turns 60. When she retires, she would like to have \$1,000,000 of savings. She is going to achieve the savings by contributing to a sinking fund between now and her retirement, with equal monthly payments paid at the end of each month. Assume the interest rate is 6% per year, compounded monthly. How much should her monthly payments be? \$701.90

29. Given the matrices A and B, perform the indicated operations or state that it's not possible. If it's not possible, explain why.

$$A=\begin{bmatrix}1 & 5 & -1\\ 2 & 3 & 0\end{bmatrix} \quad B=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & -1\\ 2 & 1 & 0\end{bmatrix}$$

(a) AB $\begin{bmatrix}3 & 0 & -4\\ 3 & 2 & -1\end{bmatrix}$

(b) BA → not possible, # cols of B ≠ # rows of A

(c) B^{-1} (You must do this by hand, using row operations—no calculators.) $\begin{bmatrix}-1 & -1 & 1\\ 2 & 2 & -1\\ -1 & -2 & 1\end{bmatrix}$

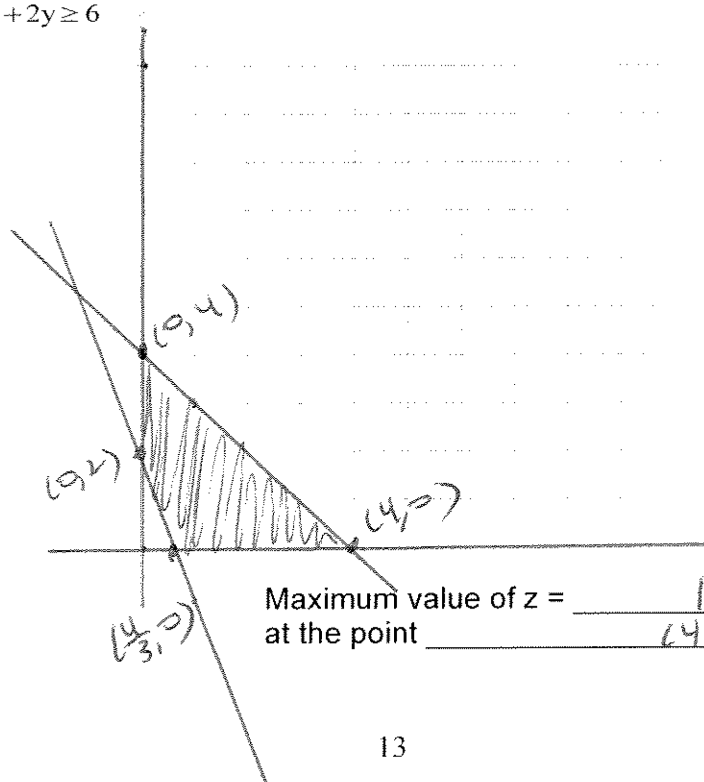
30. Determine if the system of equations below has any solutions. If a solution exists, find it. (Show all work.)

$$\begin{aligned} x + y + z &= 6\\ x + 2y + 3z &= 14\\ 2x + y + 2z &= 10\end{aligned}$$

(1,2,3)

31. Maximize the objective function z = 4x – 3y subject to the constraints:

$x \geq 0$
$y \geq 0$
$x + y \leq 4$
$3x + 2y \geq 6$



Maximum value of z = 16
at the point (4,0)