Math1090 Final Exam
Spring, 2008

Name _____ Key _____

Instructions:

- Show all work as partial credit will be given where appropriate.
- If no work is shown, there may be no credit given.
- All final answers should be written in the space provided and in simplified form.

DO *NOT* WRITE IN THIS TABLE!!!
(It is for grading purposes.)

Grade:

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Total

1) (10 pts) Find all solutions to the following equation.

$$\frac{x+2}{x+3} - \frac{1}{4} = \frac{x}{3x+9}$$

$3(x+3)$

LCD $= 4(3)(x+3)$

$$4(3)(x+3)\frac{(x+2)}{(x+3)} - \frac{1}{4}(4)(3)(x+3) = \frac{x}{3(x+3)}(4)(3)(x+3)$$

$$12(x+2) - 3(x+3) = 4x$$

$$12x + 24 - 3x - 9 = 4x$$

$$9x + 15 = 4x$$

$$15 = -5x$$

$$\boxed{-3 = x}$$

but $x = -3$ makes denominator zero in original eqn

$\Rightarrow$ N.S.

Answer 1: N.S.

2) (15 pts) Write the equation of the line through the point (-3, -1)
(a) that has undefined slope.
(b) that is perpendicular to the line $x=4y-8$.

(a) undefined slope means vertical line

$\Rightarrow$ $x=?$ + it goes thru $(-3,-1)$

$\Rightarrow$ $\boxed{x=-3}$

(b) $x+8=4y$

$\frac{1}{4}x+2=y$ this line has slope $=\frac{1}{4}$

$\Rightarrow$ $\perp$ slope is -4

$m=-4$ $(-3,-1)$

$y+1=-4(x+3)$

$y+1=-4x-12$

$\boxed{y=-4x-13}$

Answer 2(a): $x=-3$

Answer 2(b): $y=-4x-13$

3) (20 pts) A manufacturer sells cell phones for $50 each. The costs incurred in the production and sale of the cell phones are $65,000 plus $10 for each cell phone produced and sold.
(a) Write the revenue function, cost function and profit function.
(b) How many cell phones must be produced and sold to break even?

$R(x) = 50x$

$C(x) = 65000 + 10x$

$P(x) = R(x) - C(x) = 50x - (65000 + 10x)$
$= 40x - 65000$

Revenue function: $R(x) = 50x$

Cost function: $C(x) = 65000 + 10x$

Profit function: $P(x) = 40x - 65000$

$x = ?$ for $P = 0$

$40x - 65000 = 0$

$40x = 65000$

$x = \frac{65000}{40} = 1625$

cell phones sold to break even = 1625

4) (25 pts) If $f(x)=\frac{2x+1}{x+1}$ and $g(x)=3x-2$, find each of the following, simplifying as far as possible.

(a) domain of $f(x)$ $x \in \mathbb{R}, \; x \neq -1$

domain of $g(x)$ $x \in \mathbb{R}$

(b) $f(g(x)) =$ $\frac{6x-3}{3x-1}$ or $\frac{3(2x-1)}{3x-1}$

$$f(3x-2) = \frac{2(3x-2)+1}{(3x-2)+1} = \frac{6x-4+1}{3x-2+1} = \frac{6x-3}{3x-1}$$

(c) $(g \circ f)(x) =$ $\frac{4x+1}{x+1}$

$$g\left(\frac{2x+1}{x+1}\right) = 3\left(\frac{2x+1}{x+1}\right) - 2 = \frac{3(2x+1)}{x+1} - \frac{2(x+1)}{x+1} = \frac{6x+3-2x-2}{x+1} = \frac{4x+1}{x+1}$$

(d) $f^{-1}(x) =$ $\frac{1-x}{x-2}$

$$x = \frac{2y+1}{y+1} \qquad x(y+1) = 2y+1$$
$$yx + x = 2y+1$$
$$yx - 2y = 1-x$$
$$y(x-2) = 1-x \rightarrow y = \frac{1-x}{x-2}$$

(e) $g^{-1}(x) =$ $\frac{x+2}{3}$

$$x = 3y-2 \qquad x+2 = 3y$$
$$\frac{x+2}{3} = y$$

5) (20 pts) For the parabola given by equation $y=-x^2+2x+3$, answer the following questions.

(a) Find the coordinates of the vertex.

$y(1) = -1 + 2 + 3 = 4$

$a=-1 \quad b=2 \quad c=3$

$x = \frac{-b}{2a} = \frac{-2}{2(-1)} = 1$

Vertex: $(1, 4)$

(b) Find the x-intercept(s), if any. (where $y=0$)

$-1(0) = (-x^2+2x+3)(-1)$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = 3, -1$

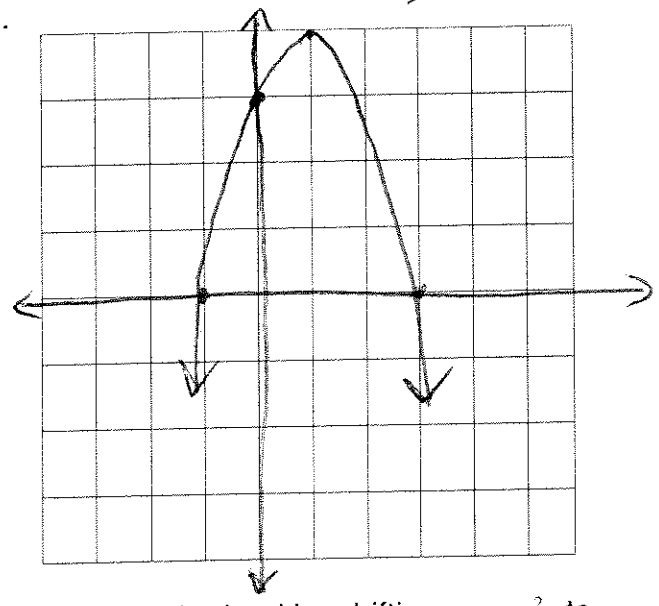
x-intercepts: $(3, 0)$ $(-1, 0)$

(c) Find the y-intercept. (where $x=0$)

$y = 0 + 2(0) + 3 = 3$

y-intercept: $(0, 3)$

(d) Sketch the graph of the parabola.



6) (5 pts) What is the equation of the parabola obtained by shifting $y=x^2$ to the left 5 units and down 2 units?

$y = (x+5)^2 - 2$

7) (20 pts) Given the matrices A, B, C and D, perform the indicated operations or state that it's not possible. If it's not possible, explain why.

$A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0 & 0\\ 1 & 4 & 2\end{bmatrix}$ and $C=\begin{bmatrix}1 & 1 & 3\\ 1 & 4 & 4\\ 2 & 2 & 7\end{bmatrix}$ and $D=\begin{bmatrix}7 & 2\\ 3 & 1\\ 2 & 3\end{bmatrix}$

(a) $-2AB$ $(2\times2)(2\times3)\rightarrow(2\times3)$

$$AB=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 1 & 4 & 2\end{bmatrix}=\begin{bmatrix}1+2 & 0+8 & 0+4\\ 0+1 & 0+4 & 0+2\end{bmatrix}$$

$$=\begin{bmatrix}3 & 8 & 4\\ 1 & 4 & 2\end{bmatrix}$$

$$-2AB=-2\begin{bmatrix}3 & 8 & 4\\ 1 & 4 & 2\end{bmatrix}$$

$-2AB =$ $\begin{bmatrix}-6 & -16 & -8\\ -2 & -8 & -4\end{bmatrix}$

(b) BA

$(2\times3)(2\times2)$

We can't multiply these together because the # of columns of B ≠ # of rows of A.

$BA =$ Not possible

(Note: This is #7 continued.)

$A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0 & 0\\ 1 & 4 & 2\end{bmatrix}$ and $C=\begin{bmatrix}1 & 1 & 3\\ 1 & 4 & 4\\ 2 & 2 & 7\end{bmatrix}$ and $D=\begin{bmatrix}7 & 2\\ 3 & 1\\ 2 & 3\end{bmatrix}$

(c) C^{-1}

$$\begin{bmatrix}1 & 1 & 3 & 1 & 0 & 0\\ 1 & 4 & 4 & 0 & 1 & 0\\ 2 & 2 & 7 & 0 & 0 & 1\end{bmatrix} \xrightarrow{(-2)(-1)} \begin{bmatrix}1 & 1 & 3 & 1 & 0 & 0\\ 0 & 3 & 1 & -1 & 1 & 0\\ 0 & 0 & 1 & -2 & 0 & 1\end{bmatrix} \xrightarrow{(-1)} \begin{bmatrix}1 & 1 & 0 & 7 & 0 & -3\\ 0 & 1 & 0 & 1/3 & 1/3 & -1/3\\ 0 & 0 & 1 & -2 & 0 & 1\end{bmatrix} \xrightarrow{(-1)} \begin{bmatrix}1 & 0 & 0 & 20/3 & -1/3 & -8/3\\ 0 & 1 & 0 & 1/3 & 1/3 & -1/3\\ 0 & 0 & 1 & -2 & 0 & 1\end{bmatrix}$$

$C^{-1}=$ $\begin{bmatrix}20/3 & -1/3 & -8/3\\ 1/3 & 1/3 & -1/3\\ -2 & 0 & 1\end{bmatrix}$

(d) $B+D^{T}$

$$\begin{bmatrix}1 & 0 & 0\\ 1 & 4 & 2\end{bmatrix}+\begin{bmatrix}7 & 3 & 2\\ 2 & 1 & 3\end{bmatrix}$$

$B+D^{T}=$ $\begin{bmatrix}8 & 3 & 2\\ 3 & 5 & 5\end{bmatrix}$

8) (20 pts) Solve the following systems of equations, if possible, using any method.

(a)
$$\begin{aligned} x+2y-z&=3\\ 3x+y&=4\\ 2x-y+z&=2 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & -1 & 3\\ 3 & 1 & 0 & 4\\ 2 & -1 & 1 & 2 \end{bmatrix} \xrightarrow{(-2)\,(-3)} \begin{bmatrix} 1 & 2 & -1 & 3\\ 0 & -5 & 3 & -5\\ 0 & -5 & 3 & -4 \end{bmatrix} \xrightarrow{(-1)} \begin{bmatrix} 1 & 2 & -1 & 3\\ 0 & -5 & 3 & -5\\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow 0 \neq 1$$

8(a) Solution: N.S.

(b)
$$\begin{aligned} x+y+z&=12\\ 2x+3y+z&=6\\ 3x+4y+4z&=20 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 & 12\\ 2 & 3 & 1 & 6\\ 3 & 4 & 4 & 20 \end{bmatrix} \xrightarrow{(-3)\,(-2)} \begin{bmatrix} 1 & 1 & 1 & 12\\ 0 & 1 & -1 & -18\\ 0 & 1 & 1 & -16 \end{bmatrix} \xrightarrow{(-1)} \begin{bmatrix} 1 & 1 & 1 & 12\\ 0 & 1 & -1 & -18\\ 0 & 0 & 2 & 2 \end{bmatrix} \Rightarrow$$

$2z = 2$, $z = 1$

$y - 1 = -18$, $y = -17$

$x + -17 + 1 = 12$, $x - 16 = 12$, $x = 28$

8(b) Solution: (28, -17, 1)

9) (15 pts) A company manufactures two types of hedge trimmers, one of which is cordless. It takes 2 hours to make a cord-type trimmer and 4 hours to make a cordless trimmer. The company has a maximum of 800 total work hours to use in manufacturing each day, and the packaging department can package at most 300 trimmers per day.

(a) Find the equations for the constraints. Then, graph the feasible region and label the corner points. (Make sure to define what your variables represent.)

(b) If the company sells the cord-type trimmer for $22 and the cordless trimmer for $45, how many of each type should it produce per day to maximize its sales?

x = # cord-type trimmers

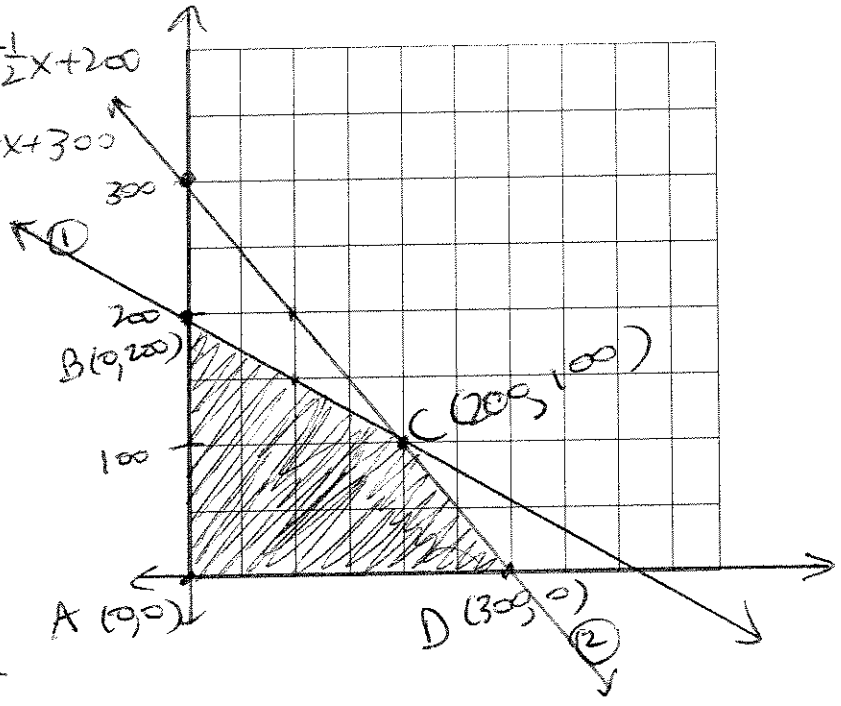
y = # cordless trimmers

$x \geq 0$

$y \geq 0$

$2x + 4y \leq 800$ ① $y \leq -\frac{1}{2}x + 200$

$x + y \leq 300$ ② $y \leq -x + 300$



C: ① & ② intersection

$-\frac{1}{2}x + 200 = -x + 300$

$\frac{1}{2}x = 100$

$x = 200$

$y = -200 + 300 = 100$

$P = 22x + 45y$

A: $P = 0$

B: $P = 0 + 45(200) = 9000$

C: $P = 22(200) + 45(100) = 8900$

D: $P = 22(300) + 0 = 6600$

For maximum, # cord-type trimmers = 0

cordless trimmers = 200

Max $ sales = $9000

10) (20 pts) Solve for x.

(a) $\dfrac{2150}{1-35e^{-3x}} = 4000$

$2150 = 4000\left(1-35e^{-3x}\right)$

$\dfrac{2150}{4000} = 1-35e^{-3x}$

$\dfrac{2150}{4000} - 1 = -35e^{-3x}$

$\dfrac{\frac{2150}{4000} - 1}{-35} = e^{-3x}$

$0.0132142857 = e^{-3x}$

$\ln(0.0132142857) = -3x$

$x = \dfrac{\ln(0.0132142857)}{-3}$

$x \approx 1.442$

$x =$ 1.442

(b) $\log_2 x + \log_2(x+2) - \log_2 3 = 4$

$\log_2\left[\dfrac{x(x+2)}{3}\right] = 4$

$2^4 = \dfrac{x(x+2)}{3}$

$16 = \dfrac{x(x+2)}{3}$

$48 = x(x+2)$

$48 = x^2+2x$

$x^2+2x-48 = 0$

$(x+8)(x-6) = 0$

$x+8=0 \quad x-6=0$

$x=-8 \quad x=6$

(can't take log of negative #)

$x =$ 6

11) (10 pts) The 5th term of an arithmetic sequence is 8 and the 8th term is 20. Find the sum of the first 100 terms of the sequence.

$a_n = a_1 + (n-1)d$ we know $a_5 = 8$, $a_8 = 20$

$\Rightarrow 8 = a_1 + 4d$

$(-1)(20 = a_1 + 7d) \Leftrightarrow$

$8 = a_1 + 4d$

$-20 = -a_1 - 7d$ +

$-12 = -3d$

$\boxed{4 = d}$

$\Rightarrow 8 = a_1 + 4(4)$

$8 = a_1 + 16$

$\boxed{-8 = a_1}$

$a_{100} = -8 + (99)4 = 388$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{100} = \frac{100}{2}(-8 + 388)$

$= 50(380) = 19000$

Sum of sequence = $\boxed{19000}$

12) (10 pts) How long does it take to pay off a car loan of $15,000, at an interest rate of 5.5% compounded monthly, if the $300 payments are due at the end of each month?

$R = \$300$ $A_n = \$15000$ $i = \frac{0.055}{12} = 0.00458\overline{3}$ $n = ?$

loan

$R = A_n\left[\frac{i}{1-(1+i)^{-n}}\right]$

$300 = 15000\left[\frac{0.00458\overline{3}}{1-1.00458\overline{3}^{-n}}\right]$

$0.02 = \frac{0.00458\overline{3}}{1-1.00458\overline{3}^{-n}}$

$0.02(1-1.00458\overline{3}^{-n}) = 0.00458\overline{3}$

$1-1.00458\overline{3}^{-n} = 0.22916$

$-1.00458\overline{3}^{-n} = -0.77083$

$\ln(1.00458\overline{3})^{-n} = \ln(0.77\ldots)$

$-n = \frac{\ln(0.77083)}{\ln(1.00458\overline{3})}$

$= 56.9$

Payoff time (in months) = about 57 months

13) (10 pts) Parents want to have $30,000 for their daughter's college fund in 18 years. If deposits are made at the end of each month, and the interest rate is 4%, compounded monthly, what is the amount of each deposit?

$FV = 30{,}000$ $\quad n = 18(12) = 216$ $\quad i = \frac{0.04}{12} = 0.00\overline{3}$

$R = ?$

$$FV = S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$30000 = R\left[\frac{1.00\overline{3}^{216} - 1}{0.00\overline{3}}\right]$$

$$30000 = 315.59248\,R$$

$$\$95.06 = R$$

Monthly deposit = $\$ 95.06$