M1060-2 QUIZ 7 (Spencer Stirling) - November 4, 2010

Directions: You may attach more sheets if necessary. SHOW ALL WORK and CLEARLY mark your solutions.

1) (4 points) Find all solutions for x in the equation

$$(\sin(2x) + \cos(2x))^2 = 0$$

LHS $(\sin(2x) + \cos(2x))^2 = \sin^2(2x) + 2\sin(2x)\cos(2x) + \cos^2(2x)$

but $\sin^2(2x) + \cos^2(2x) = 1$ (pyth. thm.)

and $2\sin(2x)\cos(2x) = \sin(2x+2x) + \sin(2x-2x)$

(product-to-sum formula)

so ~~LHS~~ equation is $1 + \sin(4x) + \underbrace{\sin(0)}_{0} = 0$

so $\sin(4x) = -1 \Rightarrow 4x = \frac{3\pi}{2} + 2\pi n$

so $\boxed{x = \frac{3\pi}{8} + \frac{\pi}{2} n}$ n = any integer

2) (4 points) Find all solutions for x in the equation

$$\tan\left(\frac{x}{2}\right) - \sin(x) = 0$$

half-angle formula says $\tan\left(\frac{x}{2}\right) = \frac{1-\cos(x)}{\sin(x)}$

so have $\frac{1-\cos(x)}{\sin(x)} = \sin(x) \Rightarrow 1 - \cos(x) = \sin^2(x)$

$\Rightarrow 1 - \cos(x) = 1 - \cos^2(x) \Rightarrow \cos^2 x = \cos(x)$

either $\cos(x) = 0$ OR $\cos(x) = 1$

$x = \frac{\pi}{2} + 2\pi n$ $\qquad x = 0 + 2\pi n = 2\pi n$

$x = \frac{3\pi}{2} + 2\pi n$

n = any integer

3) (4 points) Find all solutions for x in the equation

$$\sin(6x) + \sin(2x) = 0$$

LHS use sum to product formula

$$\sin(6x) + \sin(2x) = 2\sin\left(\frac{6x+2x}{2}\right)\cos\left(\frac{6x-2x}{2}\right)$$

$$= 2\sin(4x)\cos(2x)$$

eqn is $2\sin(4x)\cos(2x) = 0$

so either $\sin(4x) = 0$ or $\cos(2x) = 0$

$4x = 0 + 2\pi n$

and

$4x = \pi + 2\pi n$

$2x = \frac{\pi}{2} + 2\pi n$

and

$2x = \frac{3\pi}{2} + 2\pi n$

$\Downarrow$ $\Downarrow$

$x = \frac{\pi}{2} n$

and

$x = \frac{\pi}{4} + \frac{\pi}{2} n$

$x = \frac{\pi}{4} + \pi n$

and

$x = \frac{3\pi}{4} + \pi n$